Fast Adaptive Decision-Selection Equalizer Convergence Using a Tree-Structured Algorithm

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Introduction

- Adaptive decision feedback equalizers (ADFE) generally work well for digital communications but are challenged by nonlinear channels (e.g., poor grade amplifiers)
- Past research (Sebald and Bucklew [2]) considered a *selection* technique (as opposed to *feedback*) that opened the filtering construct to mildly nonlinear problems (e.g., monotonic nonlinearity)
- The drawback of the adaptive decision selection equalizer (ADSE) is slower convergence at a rate inverse to the number of possible selected hyperplanes
- This research applies a tree-structured decomposition of the linear hyperplanes to the ADSE that speeds convergence on the order of the ADFE while remaining computationally manageable
- In this sense, tree-structured ADSE is a **generalization** of the ADFE that works on mildly nonlinear channels

Block Diagram For Pulse Amplitude Modulation

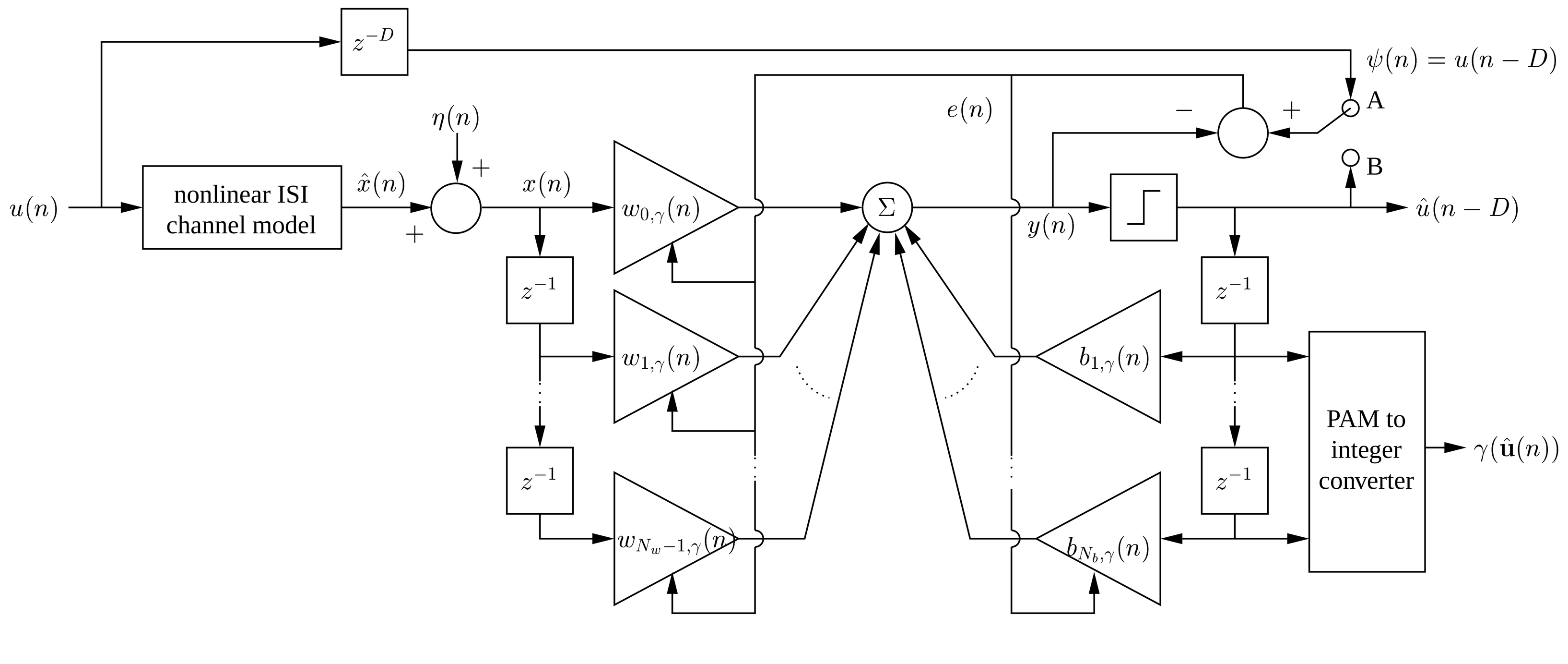

Figure 1: Block diagram of an ADSE. With switch in position A, the ADSE is in training mode. Position B corresponds to decision-directed mode.

Mathematical Definitions

Information signal

$u(n) \in \{+1, -1\}$, IID

Added noise

$\eta(n) \in \mathbb{R}$, AWGN, zero mean

Nonlinear channel output

$\hat{x}(n) \in \mathbb{R}$

Equalizer input

$x(n) = \hat{x}(n) + \eta(n)$

Feedforward coefficients

$w_{k,\gamma}(n) \in \mathbb{R}, k = 0, \ldots, N_w - 1$

Channel dispersion delay

$D \in \mathbb{Z}$

Delayed past detections

$\hat{u}(n-D)$

Feedback coefficients

$b_{k,\gamma}(n) \in \mathbb{R}, k = 1, \ldots, N_b$

Selection variable

$\gamma(n) \in \mathbb{Z}$

Vector notation

$$\mathbf{x}(n) = [x(n) \; \ldots \; x(n - N_w + 1)]^T$$
$$\mathbf{w}_\gamma(n) = [w_{0,\gamma}(n) \; \ldots \; w_{N_w-1,\gamma}(n)]^T$$
$$\hat{\mathbf{u}}(n) = [\hat{u}(n-D-1) \; \ldots \; \hat{u}(n-D-N_b)]^T$$
$$\mathbf{b}_\gamma(n) = [b_{1,\gamma}(n) \; \ldots \; b_{N_b,\gamma}(n)]^T$$

Network output

$$y(n) = \mathbf{w}_\gamma^T(n)\,\mathbf{x}(n) + \mathbf{b}_\gamma^T(n)\,\hat{\mathbf{u}}(n)$$

Sign-based detection strategy

$$\hat{u}(n-D) = \mathrm{sign}\{y(n)\}$$

Channel/Receiver Model

Simple Wiener Model [4]:

$$\tilde{x}(n) = \sum_{k=0}^{N_h-1} h_k u(n-k) = 0.4084\,u(n) + 0.8164\,u(n-1) + 0.4084\,u(n-2)$$

$$\hat{x}(n) = \sum_{p=1}^{P} c_p \tilde{x}^p(n) = \tilde{x}(n) + 0.2\,\tilde{x}^2(n) + 0.1\tilde{x}^3(n)$$

$N_w = 2$

$N_b = 2$

$D = 1$

Pattern Space

$$C_{\pm1,D,j} = \{\hat{\mathbf{x}}_k(n) | u(n-D) = \pm1, \mathbf{u}(n) = \mathbf{u}_j\}$$
$$\mathcal{H}_j = \{\mathbf{x} : \mathbf{w}_j^T(n)\,\mathbf{x} + \mathbf{b}_j^T(n)\,\mathbf{u}_j = 0\}$$

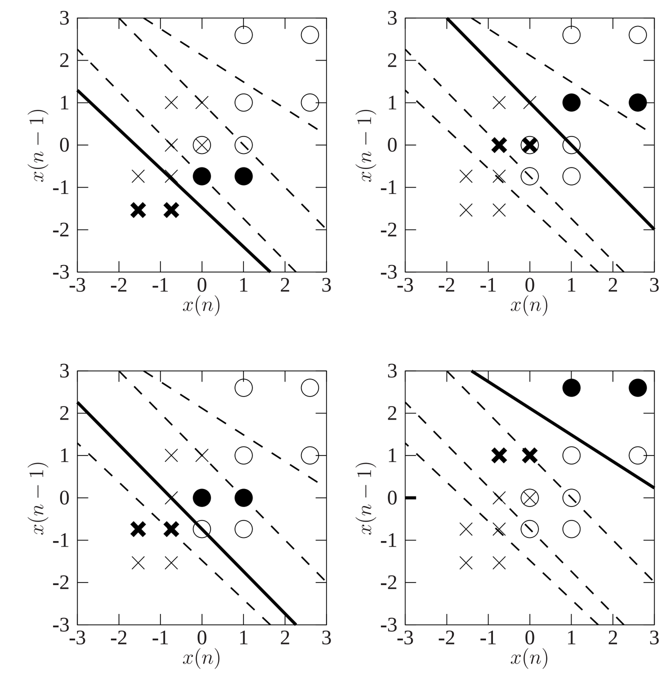

Figure 2: Conditional constellations for equalizer with $N_w = 2$, $N_b = 2$, and $D = 1$.

Tree Structure

$$\mathbf{w}_j(n) = \sum_{k=0}^{N_b} \mathbf{w}_{k,m_k}(n)$$

$$\mathbf{b}_j(n) = \sum_{k=0}^{N_b} \mathbf{b}_{k,m_k}(n)$$

$$m_k(n) = \sum_{i=1}^{k} (1 + \hat{u}(n-D-i))\, 2^{k-i-1}$$

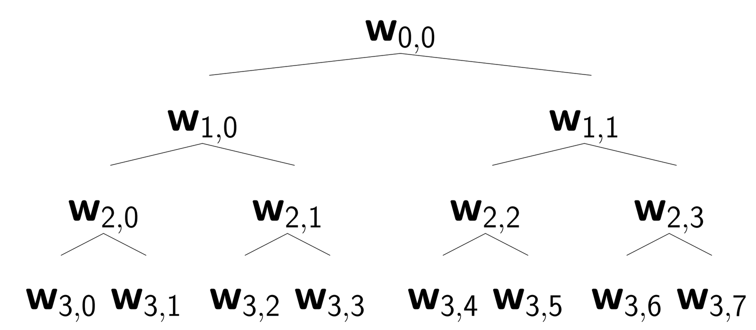

Figure 3: A sample tree structure, $N_b = 3$.

LMS-Based, Tree-Structured ADSE Algorithm

0) Initialize variables: $n = 0$,

$\mathbf{b}_{k,\ell}(0) = \mathbf{0}$, for $k = 0, \ldots, N_b$ and $\ell = 0, \ldots, 2^k - 1$,

$\mathbf{w}_{k,\ell}(0) = \mathbf{0}$, for $k = 1, \ldots, N_b$ and $\ell = 0, \ldots, 2^k - 1$,

$\mathbf{w}_{0,0}(0) = [0, \ldots, 0, 1]^T$.

1) Branch Indices: $m_0(n) = 0$ and for $k = 1 \ldots N_b$

$$m_k(n) = \sum_{i=1}^{k} (1 + \hat{u}(n-D-i))\, 2^{k-i-1}$$

2) Output and Symbol Estimate:

$$y(n) = \left(\sum_{j=0}^{N_b} \mathbf{w}_{j,m_j(n)}^T(n) \right) \mathbf{x}(n) + \left(\sum_{j=0}^{N_b} \mathbf{b}_{j,m_j(n)}^T(n) \right) \hat{\mathbf{u}} \qquad \hat{u}(n-D) = \mathrm{sign}(y(n))$$

3) Trunk Level:

$$y_0(n) = y(n)$$
$$e_0(n) = \mathrm{sign}(y_0(n)) - y_0(n)$$
$$\mathbf{w}_{0,0}(n+1) = \mathbf{w}_{0,0}(n) + \mu\, e_0(n)\, \mathbf{x}(n)$$
$$\mathbf{b}_{0,0}(n+1) = \mathbf{b}_{0,0}(n) + \mu\, e_0(n)\, \hat{\mathbf{u}}(n)$$

4) Branch Level: for $i = 1 \ldots N_b$

$$y_i(n) = \left(\sum_{k=0}^{i-1} \mathbf{w}_{k,m_k(n)}^T(n+1) + \sum_{\ell=i}^{N_b} \mathbf{w}_{\ell,m_\ell(n)}^T(n) \right) \mathbf{x}(n) + \left(\sum_{k=0}^{i-1} \mathbf{b}_{k,m_k(n)}^T(n+1) + \sum_{\ell=i}^{N_b} \mathbf{b}_{\ell,m_\ell(n)}^T(n) \right) \hat{\mathbf{u}}(n)$$

$$e_i(n) = \mathrm{sign}(y_i(n)) - y_i(n)$$
$$\mathbf{w}_{i,m_i}(n+1) = \mathbf{w}_{i,m_i}(n) + \mu\, e_i(n)\, \mathbf{x}(n)$$
$$\mathbf{b}_{i,m_i}(n+1) = \mathbf{b}_{i,m_i}(n) + \mu\, e_i(n)\, \hat{\mathbf{u}}(n)$$

5) Repeat: increment n, go to step 1.

Results

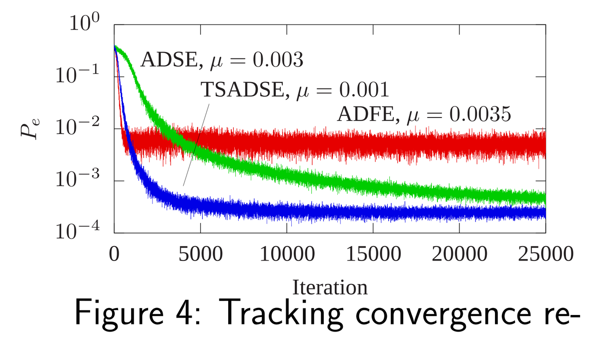

Figure 4: Tracking convergence results (500 ensemble trials) for the system described in the caption of Fig. 6 with SNR = 17 dB.

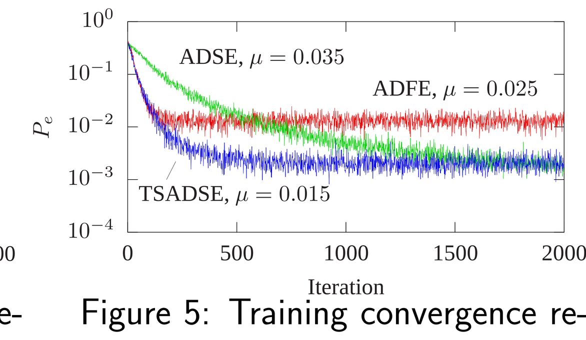

Figure 5: Training convergence results (500 ensemble trials) for the system described in the caption of Fig. 6 with SNR = 17 dB.

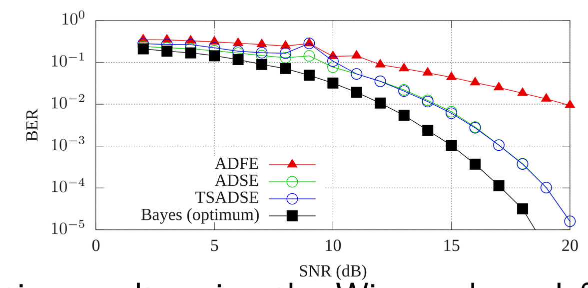

Figure 6: Simulation results using the Wiener channel for ADFE, ADSE, TSADSE, and Bayes detectors with $N_b = 4$, $N_w = 6$, $D = 5$. To speed simulations, detectors were trained for 4,000 iterations with the update constants listed in Fig. 5 and then trained another 4,000 iterations with the update constants listed in Fig. 4. Statistics were then accumulated using the update constants listed in Fig. 4. The last few low probability points have increased relative precision, *i.e.*, ratio of standard deviation to average probability, and therefore are less certain than other test samples.

Conclusions

- The ADFE may be generalized to work on nonlinear problems
  - Trunk is analogous to average orientation (ADFE)
  - Successive levels refine the orientation
  - The last level alone is the ADSE of [2]
  - May have less levels than feedback elements
- The tree-structured decomposition speeds LMS ADSE adaptation on the order of ADFE
- Misalignment is greater because of multiple levels of decomposition

Postscript: Overlooked Detail?

The misalignment issue might be rectified by propagating a correction term through non-selected branch bifurcations

References

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